

## Basic knowledge

# Vibrations

Vibration is the process occurring when a physical quantity periodically changes depending on time. This is associated with a conversion of energy from one form to another.

In the case of mechanical vibrations, periodic potential energy is converted into kinetic energy and the reverse. Every mechanical vibration is an unevenly accelerated motion. It arises due to the addition of energy to a vibratory system, e.g. a pendulum, which has force applied to it.

When the system continues to vibrate with a constant amplitude, the vibration is referred to as **undamped** motion. Without further addition of energy, each vibration is damped more or less strongly, i.e. its amplitude decreases. If the course of the vibration can be described by a sinusoidal function, we call the vibration a **harmonic** motion.

### Characteristic variables of a vibration

Characteristic variable	Formula (symbol)	Technical description
Elongation (deflection)	$y = y(t)$	Momentary distance of the vibrating body from the rest or equilibrium position
Amplitude	$\hat{y}$ oder $y_m$	Maximum value of elongation
Frequency	$f = 1/t$	Number of vibrations per unit of time $t$
Vibration period	$T = 1/f$	Duration of a complete vibration
Angular frequency	$\omega = 2 \cdot \pi \cdot f$	Angular velocity of a circular motion, whose projection results in a harmonic oscillation, indicates the exceeded phase angle of the vibration per time period
Phase angle	$\phi = \omega \cdot t + \phi_0$	Indicates the current state of a harmonically oscillating system or a shaft (either degrees or radians); a vibration period is equal to a phase angle of $2\pi$
Zero phase angle (phase constant)	$\phi_0$	Phase angle at time $t = 0$
Restoring force	$F_R$	Force that constantly pulls the vibrating body back to its rest position, which is opposite to the elongation direction
Empirical value	$k$	The proportionality factor between restoring force and elongation in elastic vibrations is identical to the spring stiffness
Natural frequency		Frequency at which the system oscillates in the natural mode after a single excitation
Damping		Removal of the amplitude in the course of a vibration

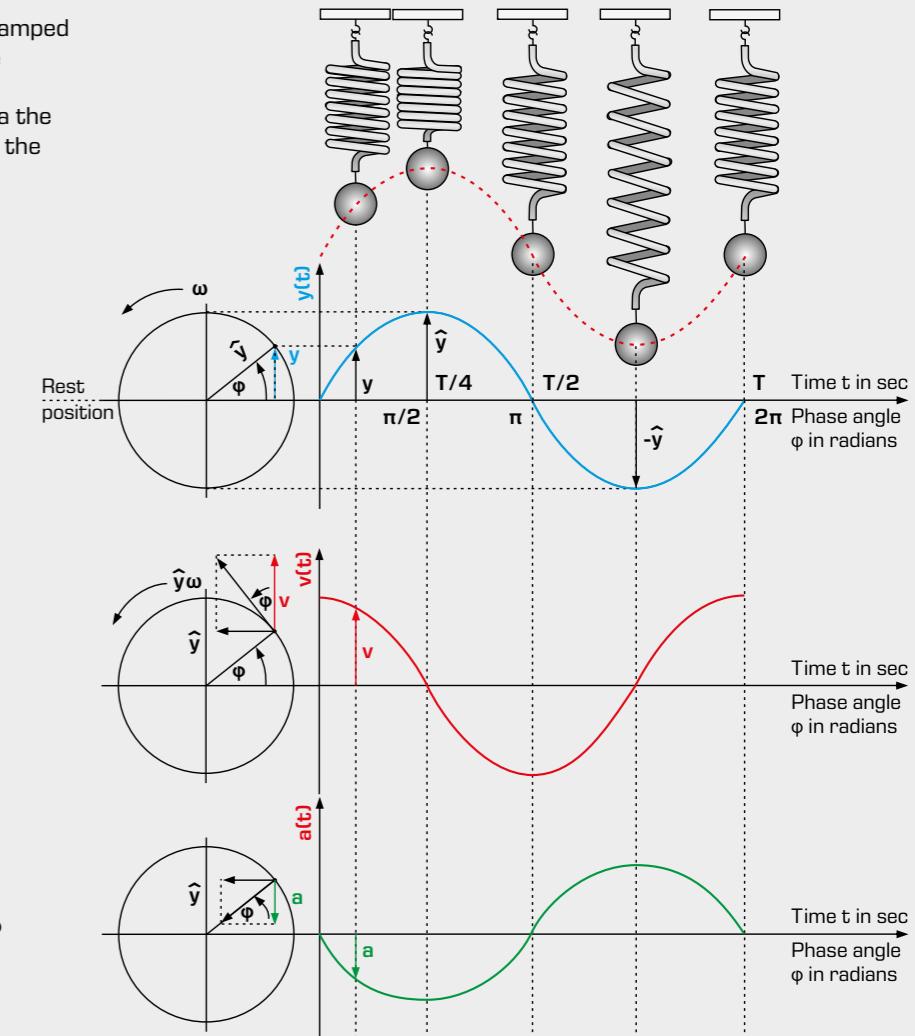
### Undamped harmonic motion

The characteristic variables of an undamped harmonic motion are illustrated in the diagram using the example of a spring pendulum. The velocity is calculated via the first derivative, the acceleration using the second derivative:

Elongation       $y = y(t)$   
 $y = \hat{y} \cdot \sin\phi$

Velocity       $v = \dot{y}$   
 $v = \hat{y} \cdot \omega \cdot \cos\phi$   
 $v = \hat{v} \cdot \cos\phi$

Acceleration       $a = \ddot{y}$   
 $a = -\hat{y} \cdot \omega^2 \cdot \sin\phi$   
 $a = -y \cdot \omega^2$



### Torsional vibration

In torsional vibration, a rotatably mounted solid body vibrates around one of its axes (rotary degree of freedom), as opposed to translational vibration. The terms "torsional" and "rotary" are synonymously used. In some applications, however, it is usual to use either one of the two terms. For example, we talk about torsional vibration when referring to a shaft twisting during a process (warping).

Every torsional vibration is made possible by a restoring moment, which is proportional to the angle of rotation at any time, but in the opposite direction.

The linear vibration laws also apply to torsional vibrations.

Elongation	$y$	$\hat{y}$	$\equiv$	Angle of rotation	$\epsilon = \hat{\epsilon} \cdot \sin\phi$
Velocity	$v = \dot{y}$	$\hat{v}$	$\equiv$	Angular velocity	$\dot{\epsilon} = \hat{\epsilon} \cdot \omega \cdot \cos\phi$
Acceleration	$a = \ddot{y}$	$\hat{a}$	$\equiv$	Angular acceleration	$\ddot{\epsilon} = \hat{\epsilon} \cdot \omega^2 \cdot \sin\phi$